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EFFECT OF THE PARAMETERS OF THE BIAXIAL FIELD OF ROCK PRESSURE ON THE SHAPE OF THE FRACTURE ZONE FORMED BY AN EXPLOSION OF A CYLINDRICAL CHARGE IN A BRITTLE MEDIUM

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The effect of the biaxial field of external rock pressure on the deformation of the fracture zone formed by radial cracks in an elastic-brittle medium is studied. We consider cylindrical charges that are rather thin compared to the diameter of the explosive borehole. This allows one to exclude the grinding zone from consideration. At the initial moment of time, the system of emerging cracks originating at the boundary of the circular hole is assumed to be symmetric. To solve the problem, we use singular integral equations and the fracture criterion σ_ϑ. *The propagation trajectories of the system cracks are calculated in the quasistatic approximation in a step-by-step manner in relation to the parameters of the external compressive stress field. Two ideal variants of loading of the crack system are analyzed. In the first variant, gaseous detonation products penetrate into cracks, and the pressure in the explosion cavity and the cracks instantaneously equalizes. In the second variant, gases do not penetrate into the cracks of the system. The fracture zone is shown to become an ellipse whose long axis is oriented in the direction of the largest compressive stress in magnitude acting at infinity. The effect of the variants of loading of the crack system on the shape and dimensions of the deformed fracture zone is evaluated.*

In describing the fracturing action of an explosion in a solid medium, use is usually made of zone fracture models [1–3] that contain three zones in the general case: a grinding zone directly adjacent to the HE charge, where the medium is considered a sand medium and the stress state in it is determined by the Coulomb law, a zone of column elasticity formed by a system of radial cracks, and an outer elastic-medium zone. As a rule, the external stress field in the zone models is given by one parameter, namely, the counter pressure; in view of this, in describing a camouflet explosion of a concentrated or cylindrical charge, the fracture zone, i.e., the zone of radial cracks, is shaped like a sphere or an infinitely long cylinder by virtue of the symmetry. It is of interest to trace the effect of the parameters of the external stress field on the shape of the fracture zone, where the external field is prescribed by stresses p and q that act at infinity in orthogonal directions. We consider the planar case, i.e., a camouflet explosion of a cylindrical charge. The cylindrical charge is assumed to have a rather small radius ($r_0 < R$, where R is the radius of the borehole). This allows us not to consider the grinding zone. Instead of the other two zones, we analyze the solution of the elastic problem of the stress state of an elastic plane with a system of smooth cuts-cracks issuing from the boundary of a circular hole. We use here the same notation as that used in [4]. We dwell briefly upon the basic aspects of problem formulation and upon the assumptions used. To derive the solution, the method of singular integral equations was employed [5, 6].

Quasistatic Formulation of the Problem. We consider an isotropic elastic plane containing N smooth curvilinear cuts issuing from a circular hole of radius R. It is assumed that uniform compressive

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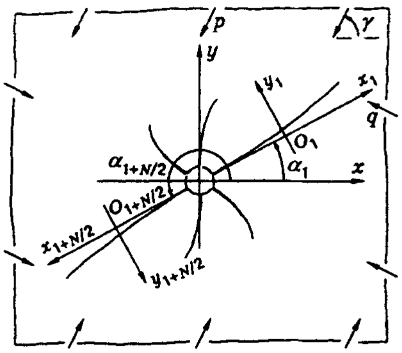

Fig. 1

stresses of intensity p and q act at infinity in two mutually orthogonal directions. Each cut L_k ($k = \overline{1,N}$) is referred to its local rectangular coordinate system $x_k O_k y_k$ (Fig. 1), and the cut's shape is given by the parametric equation

$$t_k = \omega_k(\xi) = x_k(\xi) + iy_k(\xi) \qquad (|\xi| \leqslant 1,\ t_k \in L_k)$$

in this coordinate system and by the equation $T_k(\xi) = e^{i\alpha_k}\omega_k(\xi) + z_k^0$ in the basic coordinate system, where α_k is the angle between the positive directions of the axes Ox and $O_k x_k$, $(x_k^0;\ y_k^0)$ is the origin of the local coordinate system in the basic coordinate system; here the left end of each cut is found for $\xi = -1$ and reaches the hole's boundary.

If one specifies the normal and tangential stresses

$$\sigma_{nk}^{\pm} + i\sigma_{\tau k}^{\pm} = p_k^0(t_k) \qquad (t_k \in L_k,\ k = \overline{1,N})$$

where the superscripts plus and minus refer to the upper and lower edge of the cut, the solution of the first elastic problem reduces to finding the solution $g_k'(\xi)$ of a system composed of N complex singular integral equations [6]

$$\frac{1}{2\pi}\sum_{k=1}^{N}\int_{-1}^{1}\left[R_{kn}(\xi,\eta)g_k'(\xi) + S_{kn}(\xi,\eta)\overline{g_k'(\xi)}\right]d\xi = P_n(\eta), \qquad |\eta| \leqslant 1 \quad (n = \overline{1,N}). \tag{1}$$

Here $g_k'(\xi) = g_k'(t_k)\omega_k'(\xi)$, $g_k'(t_k)$ is the derivative of the displacement jump

$$\frac{d}{dt_k}\left[(u_k + iv_k)^+ - (u_k + iv_k)^-\right] = \frac{i(1+æ)}{2\mu}\,g_k'(t_k);$$

$æ = 3 - 4\nu$, ν is the Poisson coefficient (planar deformation), and μ is the shear modulus. Expressions for the kernels of integral equations (1) and the right-hand sides of these relations are given in [4]. The function $P_n(\eta)$ depends on the shapes of the cuts, $p_n^0(\eta)$, p, q, and σ_0, where σ_0 is the pressure acting in the hole.

It is assumed that N is an even number. This limitation is of a purely technical character, because in this case, the problem possesses the property of central symmetry. Using the symmetry, one can change over to a system of $N/2$ equations similar to system (1) with the kernels

$$R_{kn}^*(\xi,\eta) = R_{kn}(T_k,T_n) - R_{kn}(-T_k,T_n),$$
$$S_{kn}^*(\xi,\eta) = S_{kn}(T_k,T_n) - S_{kn}(-T_k,T_n), \qquad k,n = \overline{1,N/2}.$$

It is known that the components of the stress tensor have a root singularity at the crack tips, and, therefore, we search for the solution of the system in the form $g_k'(\xi) = \varphi_k(\xi)/\sqrt{1-\xi^2}$. Applying Gauss quadrature formulas [5, 6] and the symmetry property, we derive the following system of $n(N/2-1)$ complex linear algebraic equations from (1):

$$\sum_{k=1}^{N/2}\sum_{i=1}^{N/2}\left[R^*_{kj}(\xi_i,\eta_m)\varphi_k(\xi_i)+S^*_{kj}(\xi_i,\eta_m)\overline{\varphi_k(\xi)}\right]=2nP_j(\eta_m)\quad (j=\overline{1,N/2},\ \ m=\overline{1,n-1}\,). \tag{2}$$

Here n determines the order of approximation of the solution, and

$$\xi_i=\cos\frac{\pi(2i-1)}{2n}\quad (i=\overline{1,n}\,),\qquad \eta_m=\cos\frac{\pi m}{n}\quad (m=\overline{1,n-1}\,)$$

are the zeros of the Chebyshev polynomials of the first and second kind $T_n(\xi)=\cos(n\cos\xi)$ and $U_{n-1}(\eta)=\sin(n\cos\eta)/\sqrt{1-\eta^2}$, respectively. Additional conditions that close system (2) are those from [6]

$$\varphi_k(-1)=0,\qquad k=\overline{1,N/2}, \tag{3}$$

which ensure finiteness of displacements at the left-hand ends of the cuts.

The major characteristics of cracking theory, which completely determine the stress field in the vicinity of the crack tip, namely, the stress intensity factors k_1 and k_2 for the singularity $(2r)^{-1/2}$ ($r/l \ll 1$, where l is the length of the crack), are given by the formula [6]

$$k_{1k}-ik_{2k}=-\sqrt{|\omega_k'(1)|}\,\frac{\varphi_k(1)}{\omega_k'(1)}, \tag{4}$$

where

$$\varphi_k(1)=-\frac{1}{n}\sum_{j=1}^{n}(-1)^j\varphi_k(\xi_j)\cot\frac{\pi(2j-1)}{4n};\quad \varphi_k(-1)=\frac{1}{n}\sum_{j=1}^{n}(-1)^{j+n}\varphi_k(\xi_j)\tan\frac{\pi(2j-1)}{4n},$$

via the solution of system (2) and (3) for the right-hand crack tip.

To construct the crack propagation trajectories, we use the fracture criterion σ_ϑ [5] according to which any crack develops in a plane the normal pressure on which is maximum. This direction is specified by the angle ϑ_*, which is reckoned from the positive direction of the tangent drawn to the crack tip and is given by the expression

$$\vartheta_*=2\arctan\frac{k_1-\sqrt{k_1^2+8k_2^2}}{4k_2}. \tag{5}$$

With the singularity $(2r)^{-1/2}$ in the expansion for $\sigma_{\vartheta\vartheta}$, in the vicinity of the crack tip at $\vartheta=\vartheta_*$, the coefficient K_1 along this direction is specified by the expression

$$K_1=\frac{1}{4}\cos^3\left(\frac{\vartheta_*}{2}\right)\left[k_1+3\sqrt{k_1^2+8k_2^2}\right], \tag{6}$$

with the use of which the ultimate-equilibrium condition is determined by the equality $K_1=K_{\mathrm{Ic}}/\sqrt{\pi}$, where K_{Ic} is the critical value of the stress intensity factor. In addition, we assume that, for $k_1 \leqslant 0$, the crack does not develop, i.e., its sides are superimposed.

The computation scheme of the quasistatic development of a system of cracks is as follows. At each step of crack propagation, the corresponding problem (2) and (3) is solved, and the stress intensity factors k_{1k}^+ and k_{2k}^+ ($k=\overline{1,N/2}\,$) are found at the tip of each crack according to formulas (4). After that, ϑ_{*k} and $K_1^{(i)}$, which allow one to establish the shape of a growing crack, are determined by formulas (5) and (6) [6, 7]. Using the formula [8]

$$v_k=\begin{cases} v_0(1-\mathrm{e}^{-\alpha_0\beta}), & \beta>0,\quad K_1>0,\qquad \alpha_0=\mathrm{const},\\ 0, & \beta\leqslant 0,\quad \beta=\dfrac{K_1}{K_{\mathrm{Ic}}}-1,\end{cases}$$

we find the velocities for each crack at a given moment of time and determine the increment step δ_k for them for a time lapse $\Delta t=h_t 2R/v_0$, where h_t is the parameter that changes the step in time, to calculate their trajectories in the form

$$\delta_k=v_k\Delta t/R=2h_t v_k/v_0,\qquad k=\overline{1,N/2}.$$

Thus, we construct quasistatic trajectories of crack growth. Quasistatics means that at every step of crack propagation only the variation of the local stress field in the vicinity of the crack tip, which is caused by crack propagation and its curving, is taken into account, and the effect of dynamic factors is ignored. A circumstance that favors this approach is the results of comparison of the calculated shapes of the crack trajectories with experimental shapes obtained in wedging of Plexiglas specimens in which the velocities of crack growth were of the order of 300 m/sec. The calculated trajectories are in satisfactory agreement with the experimental ones [9].

In implementing a step-by-step construction of crack trajectories, an algorithm for pressure recalculation in a hole is required. For gases in an explosive cavity, we adopt the law of pressure variation according to a two-chain adiabat of the Jones–Miller type [10] with adiabatic indices $\gamma_1 = 3$ and $\gamma_2 = 1.27$ in the form

$$p(V) = \begin{cases} p_{00}\left(\dfrac{V_{00}}{V}\right)^{\gamma_1}, & V \leqslant V_*, \\ p_{00}A\left(\dfrac{V_{00}}{V}\right)^{\gamma_2}, & V > V_*, \end{cases} \tag{7}$$

where $V_* = k^2 V_{00}$ and $A = k^{-3.46}$. The number k, which characterizes the point of transition from index γ_1 to γ_2, is found based on the following argument. Any industrial HE is defined by the following main parameters: $q_{\rm HE}$ is the specific energy (in kcal/kg), ρ is the density (in kg/m^3), and ξ is the gas release (in liter/kg). Some portion of the energy of an HE charge is carried away by the shock wave. We denote the portion of the charge energy that is spent on deformation of the elastic medium by η $(0 < \eta \leqslant 1)$. Equating this energy to the work of gases performed up to $V = V_k$, which corresponds to $p_k = 10^5$ Pa, per unit length of the charge, we obtain

$$\eta V_{00}\rho q_{\rm HE} = \int\limits_{V_{00}}^{V_*} p(V)\,dV + \int\limits_{V_*}^{V_k} p(V)\,dV = \frac{1}{2}\,(p_{00}\,V_{00} - p_*\,V_*) + \frac{1}{\gamma_2 - 1}\,(p_*\,V_* - p_k\,V_k), \tag{8}$$

where $p_* = p_{00}k^{-6}$, $V_* = k^2 V_{00}$, $p_k = p_{00}k^{-3.46}\xi_0^{-1.27}$, $V_k = \xi_0 V_{00}$, $\xi_0 = \xi\rho_0$, and ρ_0 is in g/cm^3.

Using these relations, we write Eq. (8) in the form

$$\eta\rho q_{\rm HE} = p_k k^{3.46}\xi_0^{1.27}\left[0.5(1 - k^{-4}) + \frac{k^{-4} - k^{-3.46}\xi_0^{-0.27}}{0.27}\right]. \tag{9}$$

After Eq. (9) is solved, we find the value of k entering the adiabat (7). Knowing k and the gas release ξ, we find $p_{00} = 10^5\, k^{3.46}\,\xi_0^{1.27}$ Pa. For example, for TNT, in accordance with the handbook [11], $q_{\rm HE} = 710$ kcal/kg, $\xi = 750$ liters/kg, and $\rho = 10^3$ kg/m^3; then we obtain from Eq. (9) $k = 1.96$ and $p_{00} = 46 \cdot 10^8$ Pa for $\eta = 1$ and $k = 1.29$ and $p_{00} = 10.8 \cdot 10^8$ Pa for $\eta = 0.5$.

Basheev et al. [4] considered two limiting variants: the absence of tamping and immobile tamping, which corresponds to the charge length $L_z \to \infty$. Gas outflow is taken into account in the quasisteady approximation. Two ideal cases were studied:

(1) detonation products do not penetrate into cracks, i.e., the normal stresses at the boundary of a circular hole are $\sigma_{rr} = -p(V)$, where $p(V)$ is determined by equality (7);

(2) an instantaneous redistribution of the gas pressure in the cavity and the cracks according to the adiabat occurs.

In the second case, where gases penetrate into cracks at $\sigma_0 = p_k^0 = p^0$ $(k = \overline{1, N/2})$, we use the following approximating formulas for the crack volume per unit length of the charge:

$$V = \pi R^2\,\frac{4(1-\nu^2)}{E}\,\frac{2}{N}\sum_{k=1}^{N/2}(p^0 + P_{0k})L_k(b_1 + b_2 L_k), \quad P_{0k} = \frac{p+q}{2} - \frac{p-q}{2}\cos(\pi - 2\alpha_k), \tag{10}$$

where E is the elasticity modulus of the medium, $L_k = l_k/R$ is the dimensionless length of the kth crack. b_1 and b_2 are coefficients that depend on the number of cracks (their values are given in [4]), and α_k is the angle of inclination of the chord joining the ends of the kth crack to the Ox axis. Formula (10) was derived under the assumption that the cracks are rectilinear and have the same length. In calculations, we used the

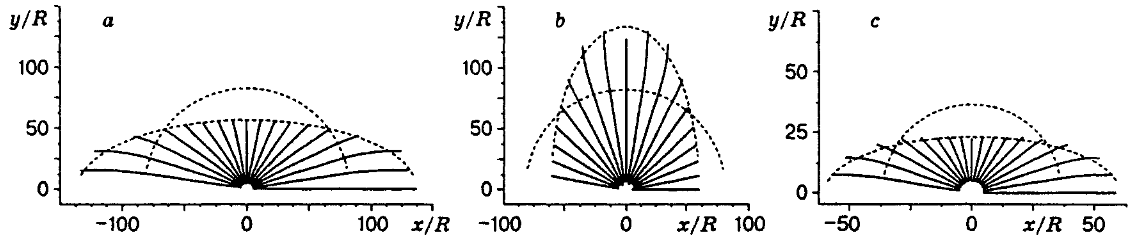

Fig. 2

length of the chord that joins the cracks' ends as the length of a crack. In addition, if any crack from the system stopped, the calculation continued with the initially chosen N, i.e., the coefficients b_1 and b_2 remained unchanged. A detailed description of the algorithm for pressure recalculation and the necessary formulas are given in [4]. We note that in the case of symmetry ($p = q$), the size of the radial-crack zone in the first case is approximately one order of magnitude smaller than in the second case, where gases penetrate into cracks. However, it is easier to reach the required accuracy in the second case, i.e., the solution converges faster to an accurate solution as the number of nodal points n increases, which is associated with the fact that the load is equal along the crack length.

Results of Calculations of Crack-Zone Deformation and Analysis. We assume that after the charge detonates, the original system containing an even number of rectilinear incipient cracks of length $l_0 = 2R$ is formed on the borehole walls. Thus, at the initial moment of time there is a system of N rectilinear cracks that are uniformly distributed over a circle. The position of this system relative to the compressive stresses at infinity is determined by the angle α_1 (Fig. 1). Gases are assumed to penetrate into cracks. TNT served as an HE, and half of the HE energy was assumed to be carried away by the shock wave.

In numerical calculations, we used data that are characteristic of sandstone and the following initial parameters: $\sigma_s = 10^8$ Pa, which is the limit of compressive strength, $E = 3 \cdot 10^{10}$ Pa, $\nu = 0.3$, $K_{\mathrm{Ic}} = 3\,\mathrm{MPa{\cdot}m^{1/2}}$, $R = 0.05$ m, $\alpha_0 = 1$, $v_0 = 650$ m/sec, $\rho = 10^3$ kg/m^3, $r_0 = 0.019$ m, $L_z = 5$ m, $q_{\mathrm{HE}} = 710$ kcal/kg, $\xi = 750$ liters/kg, $\eta = 0.5$, $k = 1.29$, and $p_{00} = 10.8 \cdot 10^8$ Pa. In this formulation, this choice of the parameters of the charge radius r_0 and its length allows one to calculate with sufficient accuracy the motion of cracks until their stoppage. Calculations were performed for $N = 2$–10. Below, we present results that correspond to $N = 6$. If $p = q = q_0$, the cracks propagate rectilinearly by virtue of the symmetry. We present values of R_0, i.e., finite dimensions of the radial-crack zone that were derived for various values of the counter pressure q_0:

$$
\begin{aligned}
-q_0 &= 5.0, \quad 10.0, \quad 20.0, \quad 25.0 \quad \text{MPa},\\
R_0/R &= 138, \quad 82.3, \quad 36.5, \quad 22.0.
\end{aligned}
$$

To illustrate how the system of cracks changes the shape of the fracture zone, we give thorough calculations for $N = 6$ for various p and q that are different in magnitude from the chosen values of p by the amount $\pm 10\,\%$ and $\alpha_1 = 0$ and $\alpha_1 = \pi/6$. This made it possible to obtain the maximum dimensions of the crack zone a_0 and b_0 for various p and q in two characteristic mutually perpendicular directions. The final dimensions of such zones depend on the number of cracks, which is rather large in these zones under real conditions, but the qualitative behavior of the deformation of the fracture zone by the system of cracks is described by the presented calculations in relation to the ratio q/p, because the basic special features of formation of a crack zone remain the same for N varying from 2 to 10.

Figure 2a shows crack trajectories for $p = -10$ MPa, $q = -10.7$ MPa, and $\alpha_1 = 0$, 10, 20, 30, 40, and 50° (solid curves) and a section of a circle of radius $R_0/R = 82.3$ and a segment of an ellipse with semi-axes $a_0/R = 136$ and $b_0/R = 56.5$ (dashed curves). Figure 2b shows similar trajectories for $p = -10$ MPa, $q = -9.5$ MPa, $a_0/R = 61$, and $b_0/R = 134.5$. It is seen that, for $p \neq q$, the zone fractured by a system of cracks can be bounded by an ellipse that is completely determined by the parameters a_0 and b_0, because if the initial system of cracks is rotated by any angle α_1, the cracks are inscribed fairly exactly in this ellipse. A similar picture is shown in Fig. 2c, where crack trajectories are calculated for $p = -20$ MPa and $q = -21$ MPa and

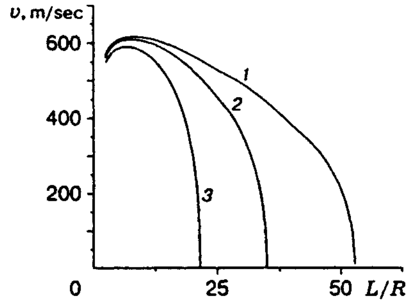

Fig. 3

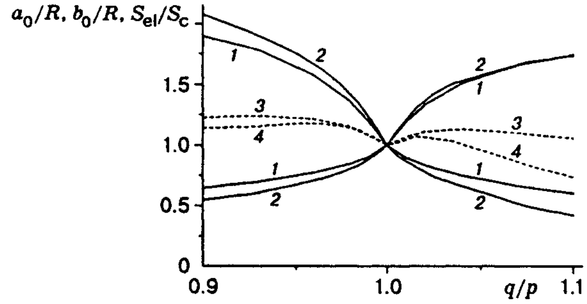

Fig. 4

TABLE 1

ξ, liter/kg	p_{00}, MPa	k	R_1/R	R_2/R	R_3/R
700	1872	1.42	76.0	30.5	14.6
750	1082	1.29	82.3	36.5	22.0
800	635	1.08	88.5	41.5	27.7

$R_0/R = 36.5$, $a_0/R = 60$, and $b_0/R = 23$. Indeed, as was noted in [4], positioned in the field of compressive stresses, this system of cracks tends to propagate in the direction of the largest compressive stress in magnitude acting at infinity, slows down, and traverses shorter distances in the perpendicular direction.

Figure 3 shows a typical dependence of the variation of the velocities of the cracks in the system on their length for $p = -20$ MPa, $q = -21$ MPa, and $\alpha_1 = 20°$. Curves 1–3 correspond to the crack number counted off counterclockwise from the horizontal axis. In the initial period of motion, the cracks seemingly speed up, and this is characteristic of all the calculations where the gas enters the cracks.

Figure 4 shows basic calculation results for the parameters of the fracture ellipse a_0/R and b_0/R, in relation to the ratio q/p (curves 1 and 2 refer to $p = -10$ and -20 MPa, respectively). We note that, for $q/p \approx 1.1$, the ratio $a_0/R \approx 1.75$, and b_0/R decreases as the absolute value of p grows; for example, as p varies from -5 to -25 MPa, $b_0/R = 0.63$–0.18. Thus, with the ratio $q/p > 1$ unchanged, the ellipse becomes narrower with increasing $|p|$, i.e., the value of b_0/a_0 decreases. In the opposite case ($q/p \approx 0.9$), as $|p|$ increases, the value of b_0/R increases weakly, whereas a_0/R decreases. In this case, with increase in the absolute value of p, the ellipse that is fractured by the system of cracks expands in the vertical direction, i.e., the ratio $b_0/a_0 \approx 3$–8 as p varies from -5 to -25 MPa. We note that a marked change in this ratio begins at $|p| > 20$ MPa. If $p = (-5)$–(-20) MPa, the ratio $b_0/a_0 = 3$–3.8 and the ratio $a_0/b_0 = 3$–4 for $q/p = 0.9$ and 1.1. Thus, there is a strong dependence of the shape of the zone fractured by the system of cracks on the relative difference in the compressive stresses acting at infinity. For example, for $(q - p)/p \approx \pm 0.1$, the circle becomes an ellipse with the ratio of semiaxes 3–4 for $p = (-5)$–(-20) MPa, where the larger semiaxis is directed toward the largest compressive stress in magnitude.

The area of the fracture ellipse S_{el} referred to the area of the circle S_c (for $p = q$) is shown as a function of q/p by curves 3 and 4 in Fig. 4 ($p = -10$ and -20 MPa). It is worth noting that, as follows from Fig. 4, the maximum volume fractured by the system of cracks does not obey the condition $p = q$, but is attained on the left and on the right of $p = q$, being larger on the left ($|q| < |p|$). For example, for $p = -10$ MPa and $q = -9.3$ MPa, the area of the ellipse is larger than the area of the circle by approximately 24% for $p = q = -10$ MPa and by 13% for $q = -10.4$ MPa. As $|p|$ increases, the values of the relative maxima decrease, and their positions approach the position of $p = q$.

Naturally, if the specific energy of the HE is retained and the gas release ξ varies, the zone fractured by radial cracks should grow with increasing ξ. Table 1 lists values of ξ, p_{00}, and k [the parameters of the adiabat (7)] found by Eq. (9) and relative radii of the radial-crack zone R_i/R ($i = 1, 2, 3$) that correspond to $p = q = -10$, -20, and -25 MPa and were calculated according to the given scheme for $q_{\rm HE} = 710$ kcal/kg

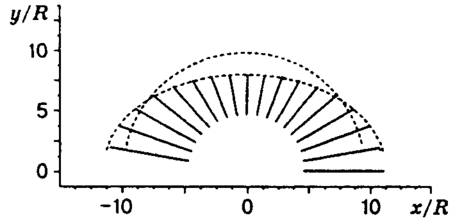

Fig. 5

TABLE 2

p, MPa	q/p	a_0/R	b_0/R	a_0/b_0	b_0/a_0
−2.50	0.80	8.5	12.7	0.67	1.49
−2.50	0.90	9.1	11.2	0.81	1.23
−2.50	1.00	9.8	9.8	1.00	1.00
−2.50	1.10	10.6	8.7	1.22	0.82
−2.50	1.20	11.2	7.9	1.42	0.70

and $\eta = 0.5$.

It follows from the very formulation of the problem that the entire deformation of the fracture zone is determined by the quantity $p-q$. One can expect that the effect of such a strong dependence of the deformation of the fracture zone on the ratio q/p is associated only with the assumption that the gases penetrate into the cracks. Indeed, cracks that propagate in a direction close to normal in relation to the direction of action of the maximum compressive stress attain the limiting equilibrium faster and slow down under the action of the external field, but this influence is not so strong. The effect of long cracks on short cracks via pressure is stronger. Long cracks have a large volume and their development rapidly decreases the pressure in the cavity, which additionally decelerates the propagation of short cracks. Therefore, long cracks will mainly develop, while the fracture zone will expand in the direction of the maximum compressive stress acting at infinity. Such strong deformation of the fracture zone occurs owing to this mechanism of gas volume redistribution.

We performed additional calculations under the assumption of gas nonpenetration into the cracks of the system to clarify the effect of gas penetration on the degree of deformation of the fracture zone. Figure 5 shows a picture of calculated crack trajectories that is similar to those depicted in Fig. 2, for $r_0 = 0.03$ m, $p = -2.5$ MPa, $q = -3.0$ MPa, and the previous values of the remaining parameters. The dashed curves refer to arcs of the circle with $R_0 = 9.8R$ ($p = q = -2.5$ MPa) and the ellipse with the semiaxes $a_0 = 11.5R$ and $b_0 = 8R$. In this variant of loading, $a_0/b_0 \approx 1.4$ if q is different from p by 20%. Table 2 shows calculation results that give an idea of the ellipse's dimensions versus the ratio of p and q.

If the gaseous detonation products do not enter the cracks, then in a first approximation where q differs comparatively little from p the final state of the deformed system of cracks is determined by the mean counter pressure, the lengths of the fracture ellipse's semiaxes vary in proportion to $|p-q|$, and an increment of the long semiaxis occurs approximately 2 times faster than for the short semiaxis.

The results obtained allow one to evaluate the degree of influence of various parameters of the problem on the shape and dimensions of the zone of fracture by the system of cracks that develop from the boundary of the explosive cavity.

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